**A Full-scale Experimental Study on Simultaneous Energy Harvesting and Vibration Control of Bridge Stay Cables using Electromagnetic Inertial Mass Dampers**

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两倍行距，新罗马字体，图和表都放在参考文献之后。

**Abstract**

**Key words**: stay cable, electromagnetic, inertial mass, circuit, energy harvesting, vibration mitigation

**1. Introduction**

1. **Description of Cable-EIMD System**

## **Configuration**

Figure 1. Configuration of cable-EIMD system for vibration control and energy harvesting

Figure 2. Configuration of prototype EIMD

## **Vibration Damping**

## **Power Flow and Efficiency**

## **3. Experimental Setup**

## **3.1 Test cable**

Figure 3. Schematic of full-scale cable vibration test setup

(a) Cable

(b) EIMD

(c) Energy harvesting circuit

(d) Data acquisition system

Figure 4. Pictures of full-scale cable vibration test setup

Table 1. Main parameters of full-scale stay cable and EIMD

## **3.2 Prototype EIMD**

## **3.3 Test Circuits**

## **3.4 Instrumentation**

## **3.5 Test Scenarios**

Table 2. Scenarios of full-scale stay cable vibration testing

## **4. Results**

## **4.1 Vibration Control Performance**

4.1.1 Damping characteristics

(a) Damping force-displacement

(b) Damping force-velocity

Figure 5. Measured damping force–displacement plot and damping force–velocity plot of the EIMD subjected to sine excitation (x Hz)

4.1.2 Free vibration response control



###### (a) displacement



(b) acceleration

Figure 6. Free vibration responses of test cable with and without EIMD

Figure 6 illustrates the time histories of the free vibration displacement and acceleration at mid-span of test cable with and without EIMD (4ton, 2.48%, R=30Ω) respectively. It can be seen that the free vibration of test cable is effectively suppressed by EIMD, in which the displacement response is reduced from 80.89mm to 19.35mm with a rate of 76.08%, while the acceleration response is reduced from 4.252m/s2 to 2.034m/s2 with a rate of 52.16%. In addition, the frequency of controlled cable with EIMD has slightly increased.



Figure 7. FFT spectra of free vibration responses of test cable with and without EIMD

Figure 7 shows the FFT spectra of the free vibration displacement at mid-span of test cable with and without EIMD (4ton, 2.48%, R=30Ω) respectively. A considerable reduction of peak in the FFT spectra (from 60.60mm to 31.18mm) due to the installation of EIMD is observed, while the corresponding frequency shifts from 1.0605Hz to 1.0986Hz accordingly.

Table 3. Modal damping ratios of test cable with and without EIMD

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Test | | Mass/ton | | Position/% | Damping ratio  (1T) | Damping ratio  (5T) | Damping ratio  (10T) |
| Without control | - | | - | | 0.0049 | 0.0010 | 0.0002 |
| With control | | 0 | | 2.48 | 0.0295 | 0.0225 | 0.0190 |
| 5 | 0.0131 | 0.0188 | 0.0158 |
| 2 | | 2.48 | 0.0515 | 0.0266 | 0.0187 |
| 5 | 0.0348 | 0.0303 | 0.0199 |
| 4 | | 2.48 | 0.0384 | 0.0366 | 0.0256 |
| 5 | 0.0556 | 0.0260 | 0.0248 |
| 6 | | 2.48 | 0.0537 | 0.0381 | 0.0240 |
| 5 | 0.0859 | 0.0287 | 0.0179 |
| 8 | | 2.48 | 0.0885 | 0.0392 | 0.0248 |
| 5 | 0.0309 | 0.0286 | 0.0233 |
| 10 | | 2.48 | 0.0256 | 0.0305 | 0.0273 |
| 5 | 0.0261 | 0.0189 | 0.0163 |

As aforementioned, a series of experiments employing EIMDs of different equivalent inertial masses (0, 2, 4, 6, 8, 10ton) and installation positions (2.48%, 5% of test cable) are conducted. The average damping ratios of test cable with and without EIMD, identified by the vibration attenuation method with the computing interval of 1, 5 and 10 vibration periods respectively, are presented in table 3. We observed that the damping ratio of the uncontrolled test cable is quite low. With the aid of EIMD, the damping ratio was enhanced to……

4.1.3 Forced vibration response control



Figure 7. Displacement time histories of test cable with and without EIMD subjected to sine sweep excitation (the first vibration mode, frequency range 0.95–1.20 Hz)

Figure 7 shows the time histories of the displacement at mid-span of test cable with and without EIMD (4ton, 5%, R=50Ω) subjected to sine sweep excitation. The frequency of sine sweep excitation ranges from 0.95Hz to 1.20Hz with the sweeping speed of 0.001Hz/s, which covers the first-order resonance frequency of test cable. Comparing with the uncontrolled test cable, a significant reduction in the displacement response can be observed by employing EIMD, and the peak displacement response is suppressed from 43.51mm to 33.47mm. It is noted that the resonant frequency in mode 1 slightly shifts from 1.085Hz to 1.146Hz, after the installation of EIMD. (explain:……)

4.1.4 Frequency response function



Figure 8. Displacement time histories of test cable with and without EIMD subjected to harmonic sweeping excitation (frequency range 0–4 Hz, 0.01Hz/s)

Figure 8 presents the time histories of displacement at mid-span of test cable under harmonic sweeping excitation with and without EIMD (4ton, 5%, R=14Ω). The frequency range (0 to 4Hz) of the harmonic sweeping excitations covers the first three modes of test cable. As is shown in this figure, the targeted modal vibration (mode 1) of test cable is suppressed from 30.67mm to 26.29mm due to the use of EIMD, along with the resonant frequency shift in mode 1. However, it is observed that the EIMD has an undesirable effect in response control of mode 2 and 3.



Figure 9. Measured FRF of displacement responses to exciting force

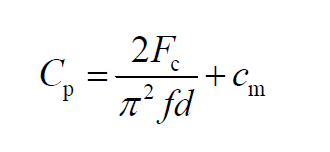
Figure 9 shows the FRFs of displacement at mid-span of test cable under harmonic sweeping excitation with and without EIMD (4ton, 5%, R=14Ω). By the installation of EIMD, the peak magnitudes of the FRF, which corresponds to the first, second and third order frequency (1.06Hz, 2.11Hz and 3.17Hz), have been considerably reduced by 12.251dB, 7.518dB and 5.370dB respectively. Notably, two dominant peaks can be seen for the controlled cable with EIMD, as well as two dominant frequencies (1.057Hz and 1.135Hz).

## **4.2 Energy Harvesting Performance**

4.2.1 Results of Resistor Case

 (x)

 (x)



According to equation (x), the average output power, which is the consumed by the load resistor, is calculated by the measured voltage across the resistance, and the average input power is calculated by the measured force and displacement of damper. Therefore, the energy harvesting efficiency could be calculated by equation (x). Figure 10 and 11 present the average output power and energy harvesting efficiency versus the load resistance respectively, in which theory, numerical (Simulink) and experiment results are included. It is observed that the average output power and energy harvesting efficiency can achieve the maximums of xW and x% correspondingly.



(a) Overall power-resistance

(b) Output power-resistance

(c) Efficiency-resistance

Figure 11. Normalized output power and energy harvesting efficiency versus load resistance **(free vibration)**

Table 4. Output power and efficiency of EIMD attached to a 135m full-scale cable

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Mass  /ton | Position  /% | Optimal Resistance | Efficiency/% | Optimal Resistance | Output power/W |
| 1.2 | 2.48 | 3 | 15.18% | 3 | 2.326545 |
| 5 | 3 | 15.97% | 3 | 1.383176 |
| 2 | 2.48 | 6 | 14.02% | 6 | 1.115996 |
| 5 | 2 | 15.28% | 2 | 1.210556 |
| 4 | 2.48 | 5 | 19.28% | 5 | 0.929619 |
| 5 | 3 | 16.10% | 8 | 0.650794 |
| 6 | 2.48 | 3 | 22.26% | 3 | 1.560024 |
| 5 | 3 | 16.88% | 5 | 1.065936 |
| 8 | 2.48 | 8 | 23.34% | 3 | 1.515854 |
| 5 | 3 | 17.65% | 3 | 2.876969 |
| 10 | 2.48 | 3 | 21.15% | 3 | 1.101988 |
| 5 | 3 | 18.78% | 3 | 1.15555 |

/\*\*\*\*\*\*\*\*\*\*其他图片\*\*\*\*\*\*/

1. free vibration & resistance case



图1等效质量与最大效率的关系（2.48%）



图2 效率与等效质量关系（5%）



图3 典型的斜拉索跨中位移频谱



图4 典型的效率理论值和测得值对比（**存在较大误差**）

2. resonant vibration & resistance case



图1等效质量与最大效率的关系（2.48%）



图2等效质量与最大效率的关系（5%）



典型的斜拉索跨中位移频谱



典型的理论值与实测值对比

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Mass  /ton | Position  /% | Optimal Resistance | Eta/% | Optimal Resistance | Pout/W |
| 0 | 2.48 | 1 | 1.27% | 1 | 0.011082 |
| 5 | 12 | 0.66% | 1 | 0.019635 |
| 2 | 2.48 | 2 | 2.99% | 1 | 0.421803 |
| 5 | 3 | 3.39% | 3 | 0.086882 |
| 4 | 2.48 | 3 | 3.73% | 1 | 0.248316 |
| 5 | 5 | 2.27% | 1 | 0.281383 |
| 6 | 2.48 | 3 | 4.20% | 1 | 0.725277 |
| 5 | 5 | 10.46% | 2 | 0.444708 |

3. harmonic sweeping & resistance case



|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| position | mass | R\_load | mode1 | | mode2 | | mode3 | | total | | |
| Pout | eta | Pout | eta | Pout | eta | Pout | Poutmax | eta |
| 2.48 | 0 | 14 | 2.46E-04 | 0.14% | 1.69E-05 | 0.01% | 9.85E-07 | 0.00% | 8.80E-05 | 0.002294 | 0.05% |
| 22 | 1.88E-05 | 0.02% | 9.41E-05 | 0.04% | 2.43E-05 | 0.01% | 4.57E-05 | 0.001062 | 0.02% |
| 2 | 14 | 0.001158 | 0.45% | 0.001676 | 0.43% | 6.14E-04 | 0.16% | 1.15E-03 | 0.017324 | 0.35% |
| 22 | 8.64E-04 | 0.39% | 0.001355 | 0.34% | 5.18E-04 | 0.12% | 9.12E-04 | 0.016851 | 0.28% |
| 4 | 14 | 9.93E-04 | 0.31% | 9.41E-04 | 0.29% | 8.08E-04 | 0.18% | 9.14E-04 | 0.013565 | 0.26% |
| 22 | 0.001611 | 0.35% | 0.001814 | 0.40% | 6.79E-04 | 0.17% | 1.37E-03 | 0.024106 | 0.31% |
| 5 | 0 | 14 | 0.002227 | 0.99% | 0.004517 | 1.13% | 0.002916 | 0.70% | 3.22E-03 | 0.054706 | 0.94% |
| 22 | 0.001863 | 0.54% | 0.002658 | 0.64% | 0.001701 | 0.47% | 2.07E-03 | 0.027957 | 0.55% |
| 2 | 14 | 8.88E-04 | 0.44% | 0.002933 | 0.58% | 0.001257 | 0.30% | 1.69E-03 | 0.025359 | 0.44% |
| 22 | 7.50E-04 | 0.35% | 0.002174 | 0.42% | 5.41E-04 | 0.16% | 1.16E-03 | 0.020528 | 0.31% |
| 4 | 14 | 0.005545 | 1.58% | 0.009162 | 1.58% | 0.00469 | 1.09% | 6.47E-03 | 0.073582 | 1.42% |
| 28 | 0.002742 | 0.82% | 0.005721 | 1.01% | 0.004279 | 0.72% | 4.25E-03 | 0.047742 | 0.85% |



## **5. Discussions**

##### On the optimal damping coefficient for EIMD design

##### On the maximum achievable modal damping ratio

##### On the energy dissipation capability of EIMD

## **6. Conclusions**

## **Acknowledgement**

The authors are grateful for the financial support from the National Natural Science Foundation of China (Grant Nos.: 51508217 & 51178203).

## **Appendix**

Formulation of cable-EIMD system and its complex eigenvalue analysis

## **References**

# **Figures**

**Figure 1.** Configuration of EMDEH for stay cable vibration control and energy harvesting

# **Tables**

**Table 1.** Main parameters of test stay cable

|  |  |  |  |
| --- | --- | --- | --- |
| Item | Value | Item | Value |
| Mass per unit length, m | 0.442 kg/m | Diameter | 4 mm |
| Cable length, *l* | 5.85 m | Cross-sectional area | 7.28 mm2 |
| Inclination | 15.5° | Young’s modulus |  |
| Static tension force | 980 N | Axial stiffness |  |